Pythagorean Theorem

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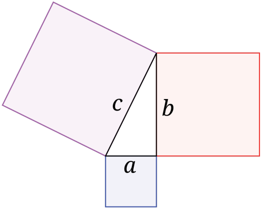
Pythagorean Theorem

In mathematics, the Pythagorean theorem, also known as Pythagoras' theorem, is a fundamental relation in Euclidean geometry among the three sides of a right triangle. It states that the area of the square whose side is the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares on the other two sides. This theorem can be written as an equation relating the lengths of the sides a, b and c, often called the "Pythagorean equation"
$a^2 + b^2 = c^2$ where c represents the length of the hypotenuse and a and b



the lengths of the triangle's other two sides

Pythagorean triples

A Pythagorean triple has three positive integers a, b, and c. In other words, a Pythagorean triple represents the lengths of the sides of a right triangle where all three sides have integer lengths. Such a triple is commonly written (a, b, c). Some well-known examples are (3, 4, 5) and (5, 12, 13).

A primitive Pythagorean triple is one in which a, b and c are coprime (the greatest common divisor of a, b and c is 1).

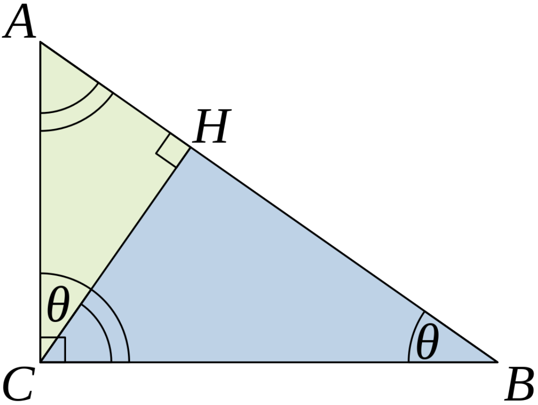
The following is a list of primitive Pythagorean triples with values less than 100:

(3, 4, 5), (5, 12, 13), (7, 24, 25), (8, 15, 17), (9, 40, 41), (11, 60, 61), (12, 35, 37), (13, 84, 85), (16, 63, 65), (20, 21, 29), (28, 45, 53), (33, 56, 65), (36, 77, 85), (39, 80, 89), (48, 55, 73), (65, 72, 97)

Proof of Pythagorean theorem

This proof is based on the proportionality of the sides of two similar triangles, that is, upon the fact that the ratio of any two corresponding sides of similar triangles is the same regardless of the size of the triangles. Let ABC represent a right triangle, with the right angle located at C, as shown on the figure. Draw the altitude from point C, and call H its intersection with the side AB. Point H divides the length of the hypotenuse c into parts d and e. The new triangle ACH is similar to triangle ABC, because they both have a right angle (by definition of the altitude), and they share the angle at A, meaning that the third angle will be the same in both triangles as well, marked as θ in the figure. By a similar reasoning, the triangle CBH is also similar to ABC. The proof of similarity of the triangles requires the triangle postulate: the sum of the angles in a triangle is two right angles, and is equivalent to the parallel postulate. Similarity of the triangles leads to the equality of ratios of corresponding sides:

$\frac{BC}{AB} = \frac{BH}{BC}$ and $\frac{AC}{AB} = \frac{AH}{AC}$. The first result equates the cosines of the angles θ, whereas the second result equates their sines.


A
H
θ
θ
C
B

Converse of Pythagorean theorem

The converse of the theorem is also true:
For any three positive numbers a, b, and c such that $a^2 + b^2 = c^2$ there exists a triangle with sides a, b and c,if $a^2 + b^2 = c^2$ and every such triangle has a right angle between the sides of lengths a and b.
An alternative statement is:
For any triangle with sides a, b, c, then the angle between a and b measures 90°